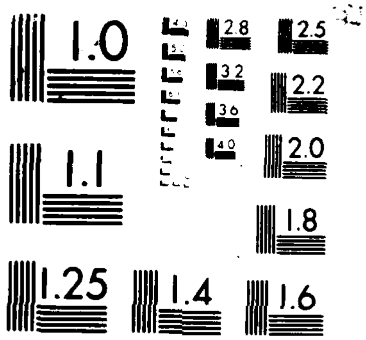
AD-A177 317 DEVELOPMENT AND APPLICATION OF THE P-VERSION OF THE 1/1
FINITE ELEMENT METHOD. (U) WASHINGTON UNIV ST LOUIS MO
DEPT OF SYSTEMS SCIENCE AND MATHE.. I N KATZ 29 OCT 86
UNCLASSIFIED SSM-8609 AFOSR-TR-87-0231 AFOSR-82-0315 F/G 12/1 NL



MICROCOPY RESOLUTION TEST CHART

AD-A177 317

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT: Distribution of this report is unlimited
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S): 8609	5. MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 87-0231
6a. NAME OF PERFORMING ORGANIZATION: Department of Systems Science and Mathematics; 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION: AFOSR/PKZA
6c. ADDRESS (City, State, and ZIP Code): Washington University, Campus Box 1040, St. Louis, MO 63130	7b. ADDRESS (City, State, and ZIP Code): Building 410, Bolling Air Force Base, DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR/NM; 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: AFOSR-82-0315 C
8c. ADDRESS (City, State, and ZIP Code): Building 410, Bolling AFB, DC 20332-6448	10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
61102F	2304	A 3	

DTIC SELECTE MAR 0 2 1987 S D

11. TITLE (Include Security Classification)

DEVELOPMENT AND APPLICATION OF THE P-VERSION OF THE FINITE ELEMENT METHOD

12. PERSONAL AUTHOR(S) I. Norman Katz

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Annual Scientific	FROM 85/10/30 TO 86/10/30	86/10/29	8

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The p-version of the finite element method is a new, important, computationally efficient, approach to finite element analysis. It is more robust than the conventional h-version and its rate of convergence, for domains with corners and for other singularity problems, is twice that of the h-version.

Hierarchic elements which implement the p-version efficiently have been formulated so as to enforce C^0 or C^1 continuity in the planar case, and so as to enforce C^0 continuity in three dimensions.

* Continued on the reverse side.

DTIC FILE COPY

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE (Include Area Code); 22c. OFFICE SYMBOL

DD FORM 1473, 84 MAR 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

Recent research accomplishments include:

1. Development of an algorithm that finds all roots of an analytic function in a finite domain.

2. Preprocessing procedures to restrict unbounded domains which contain roots to bounded ones.

3. A reliable numerical argument principle algorithm to compute number of zeros within a closed contour.

4. Formulation of equations which determine the nature of stress singularity at a corner of a plate composed of n estropic materials.

All of the above are used in the extraction method for p-version finite element analysis of composite materials with corners.

AFOSR-TR- 87-0231

CONTENTS

1. DISCUSSION

There are now three basic approaches to finite element analysis. In all approaches the domain Ω is divided into simple convex subdomains (usually triangles or rectangles in two dimensions, and tetrahedra or bricks in three dimensions) and over each subdomain the unknown is approximated by a (local) basis function (usually a polynomial of degree $\leq p$). Basis functions are required to meet continuously at boundaries of subdomains in the case of planar or 3 dimensional elasticity, or smoothly in the case of plate bending. The approaches are:

1. The <u>h-version</u> of the finite element method. In this approach the degree p of the approximating polynomial is kept fixed, usually at some low number such as 2 or 3. Convergence is achieved by allowing h, the maximum diameter of the convex subdomains, to go to zero. Estimates for the error in energy have long been known. In all of these estimates p is assumed to be fixed and the error estimate is asymptotic in h, as h goes to zero.
2. The <u>p-version</u> of the finite element method. In this approach the subdivision of the domain Ω is kept fixed but p is allowed to increase until a desired accuracy is attained. The p-version is reminiscent of the Ritz method for solving partial differential equations but with a crucial distinction between the two methods. In the Ritz method a single polynomial approximation is used over the <u>entire</u> domain Ω (Ω, in general, is not convex). In the p-version of the finite element method polynomials are used as approximations over <u>convex subdomains</u>. This critical difference gives the p-version a more rapid rate of convergence than either the Ritz method or the h-version.

3. The h-p version of the finite element method. In this approach both the degree p of the approximating polynomial and the maximum diameter h of the convex subdomains are allowed to change.

The p-version of the finite element method requires families of polynomials of arbitrary degree p defined over different geometric shapes. Polynomials defined over neighboring elements join either continuously (are in C^0) for planar or three dimensional elasticity, and smoothly (are in C^1) for plate bending. In order to implement the p-version efficiently on the computer, these families should have the property that computations performed for an approximation of degree p are re-usable for computations performed for the next approximation of degree p+1. We call families possessing this property <u>hierarchic</u> families of finite elements.

The h-version of the finite element method has been the subject of intensive study since the early 1950's and perhaps even earlier. Study of the p-version of the finite element method, on the other hand, began at Washington University in St. Louis in the early 1970's and led to a more recent study of the h-p version. Research in the p-version (formerly called The Constraint Method) has been supported in part of the Air Force Office of Scientific Research since 1976.

Recent Research Accomplishments include:

1. Development of an algorithm that finds all roots of an analytic function in a bounded domain.

2. A preprocessing procedure which finds a bounded subdomain of a given unbounded domain in which the roots of an analytic function are to be found.

3. A numerical argument principle which computes the number of zeros of an analytic function inside a closed contour. The crucial part of

this algorithm is a test to determine whether the argument change between two points on the contour is less than π in absolute value.

4. Explicit formulation of the equations which determine the nature of the stress singularity in a plate wedge composed of n isotropic materials meeting at a corner. The boundary conditions are either clamped or free.

All of the above are needed to use the extraction techniques developed earlier for the finite element analysis by the p-version of a composite material with corners.

PROFESSIONAL PERSONNEL

1. I. Norman Katz, Professor of Applied Mathematics and Systems Science, Washington University

2. Barna A. Szabo, A.P. Greensfelder Professor Civil Engineering, Washington University

3. Xing-Ren Ying, Research Assistant, doctoral candidate in the Department of Systems Science and Mathematics, Washington University

PAPERS PUBLISHED AND PRESENTED SINCE THE START OF THE PROJECT (1977)

Published Papers:

1. "Hierarchal Finite Elements and Precomputed Arrays", by Mark P. Rossow and I. Norman Katz, Int. J. for Num. Method in Engr., Vol. 13, No. 6 (1978) pp. 977-999.

2. "Nodal Variables for Complete Conforming Finite Elements of Aribtrary Polynomial Order", by I. Norman Katz, A.G. Peano, and Mark P. Rossow, Computers and Mathematics and Applications, Vol. 4, No. 2, (1978), pp. 85-112.

3. "Hierarchic Solid Elements for the p-version of the Finite Element Method", by I. Norman Katz, B.A. Szabo and A.G. Peano (in preparation).

4. "P-convergence Finite Element Approximations in Linear Elastic Fracture Mechanics", by Anil K. Mehta (doctoral dissertation), Department of Civil Engineering, Washington University (1978).

5. "An Improved p-version Finite Element Algorithm and a Convergence Result for the p-version", by Anthony G. Kassos, Jr. (doctoral dissertation) Department of Systems Science and Mathematics, Washington University, (August, 1979).

6. "Hierarchic Families for the p-version of the Finite Element Method", I. Babuska, I.N. Katz and B.A. Szabo, invited paper presented at the Third IMACS International Symposium on Computer Methods for Partial Differential Equations, published in Advances in Computer Methods for Partial Differential Equations - III (1979) pp. 278-286.

7. "The p-version of the Finite Element Method", I. Babuska, B.A. Szabo, and I.N. Katz, SIAM J. of Numerical Analysis, Vol. 18, No. 3, June 1981, pp. 515-545.

8. "Hierarchic Triangular Elements with one Curved Side for the p-version of the Finite Element Method", by I. Norman Katz (in preparation).

9. "The p-version of the Finite Element Method for Problems Requiring C^1-Continuity", Douglas W. Wang (doctoral dissertation), Department of Systems Science and Mathematics, Washington University, August 1982.

10. "Implementation of a C^1 Triangular Element based on the p-version of the Finite Element Method", by I. Norman Katz, D.W. Wang and B. Szabo, Proceedings of the Symposium in Advances and Tends in Structural and Solid Mechanics, October 4-7, 1982, Washington, D.C.

11. "The p-version of the Finite Element Method for Problems Requiring C^1-Continuity", by I. Norman Katz and Douglas W. Wang, SIAM J. of Numerical Analysis, Vol. 22, No. 6, December 1985, pp. 1082-1106.

12. "Implementation of a C^1-triangular Element based on the p-version of the Finite Element Method", by Doulgas W. Wang, I. Norman Katz and Barna A. Szabo, Computers and Structures, Vol. 19, No. 3, pp. 381-392 (1984).

13. "H- and p-version analysis of a Rhombic Plate", by Douglas W. Wang, I. Norman Katz and Barna A. Szabo, International Journal for Numerical Methods in Engineering, Vol. 20, pp. 1399-1405 (1984).

14. "Implementation of a Finite Element Software System with H and P Extension Capabilities," by Barna A. Szabo, Proceedings of the 8th Invitational Symposium on the Unification of Finite Element-Finite Differences and the Calculus of Variations.

15. "Computation of Stress Field Parameters in Areas of Steep Stress Gradients," by Barna A. Szabo, to appear in Communications in Applied Numerical Methods.

16. "On Stress Analysis with Large Length Ratios", by Barna A. Szabo, submitted to AIAA Journal.

17. "Mesh Design for the p-version of the Finite Element Method", by Barna A. Szabo, Computer Methods in Applied Mechanics and Engineering 55 (1986), pp. 181-197.

Presented Papers:

1. "Hierarchical Approximation in Finite Element Analysis", by I. Norman Katz, International Symposium on Innovative Numerical Analysis in Applied Engineering Science, Versailles, France, May 23-27, 1977.

2. "Efficient Generation of Hierarchal Finite Elements Through the Use of Precomputed Arrays", by M.P. Rossow and I.N. Katz, Second Annual ASCE Engineering Mechanics Division Speciality Conference, North Carolina State University, Raleigh, NC, May 23-25, 1977.

3. "C^1 Triangular Elements of Arbitrary Polynomial Order Containing Corrective Rational Functions", by I. Norman Katz, SIAM 1977 National Meeting, Phildelphia, PA, June 13-15, 1977.

4. "Hierarchical Complete Conforming Tetrahedral Elements of Arbitrary Polynomial Order", by I. Norman Katz, presented at SIAM 1977 Fall Meeting, Albuquerque, NM, October 31-November 2, 1977.

5. "A Hierarchical Family of Complete Conforming Prismatic Finite Elements of Arbitrary Polynomial Order", by I. Norman Katz, presented at SIAM 1978 National Meeting, Madison, WI, May 24-26, 1978.

6. "Comparative Rates of h- and p- Convergence in the Finite Element Analysis of a Model Bar Problem", by I. Norman Katz, presented at the SIAM 1978 Fall Meeting, Knoxville, Tennessee, October 20 - November 1, 1978.

7. "Smooth Approximation to a Function in $H_0^2(D)$ by Modified Bernstein Polynomials over Triangles", by A.G. Kassos, Jr. and I.N. Katz, presented at the SIAM 1979 Fall Meeting, Denver, Colorado, November 12-14, 1979.

8. "Triangles with one Curved Side for the p-version of the Finite Element Method", by I. Norman Katz, presented at the SIAM 1980 Spring Meeting, Alexandra, VA June 5-7, 1980.

9. "Hierarchic Square Pyramidal Elements for the p-version of the Finite Element Method", by I. Norman Katz, presented at the SIAM 1980 Fall Meeting, Houston, TX, November 6-8, 1980.

10. "The Rate of Convergence of the p-version of the Finite Element Method for Plate Bending Problems", by Douglas W. Wang and I. Norman Katz, presented at SIAM 1981 Fall Meeting, October 6-8, 1981, Cincinatti, Ohio.

11. "The p-version of the Finite Element Method", by I. Norman Katz, 1982 Meeting of the Illinois Section of the Mathematical Association of America, Southern Illinois University at Edwardsville, April 30-May 1, 1982.

12. "Computer Implementation of a C^1 Triangular Element based on the p-version of the Finite Element Method", by Douglas W. Wang and I. Norman Katz, SIAM 30th Anniversary Meeting, July 19-23, 1982, Stanford, California.

13. "Implementation of a C^1 Triangular Element Based on the p-version of the Finite Element Method", Symposium on Advanced and Trends in Structural and Solid Mechanics, October 4-7, 1982, Washington, D.C.

14. "P-Convergent Polynomial Approximations in $H_0^2(\Omega)$", by Douglas W. Wang and I. Norman Katz, Fourth Texas Symposium on Approximation Theory Department of Mathematics, Texas A&M University, College Station, Texas 77843, January 17-21, 1983.

15. "Design Aspects of Adaptive Finite Element Codes", by D.W. Wang, I.N. Katz and M.Z. Qian, ASCE-EMD (American Society of Civil Engineers-Engineering Mechanics Division) Speciality Conference, Purdue University, May 25-28, 1983.

16. "Smoothing Stresses Computed Pointwise by the p-version of the Finite Element Method", by I. Norman Katz and Xing-ren Ying, SIAM 1983 National Meeting, Denver, Colorado, June 6-8, 1983.

17. "The Use of High Order Polynomials in the Numerical Solution of Partial Differential Equations", a Mini Symposium, I. Norman Katz, Organizer and Chairman; "The h-p version of the Finite Element Method", I. Babuska, B. Szabo, K. Izadpanah, W. Gui, and B. Guo; "A Pseudospectral Legendre Method for Hyperbolic Equations", D. Gottlieb and H. Tal-Ezer; "The Approximation Theory for the p-version of the Finite Element Method", Milo Dorr; "On the Robustness of Higher Order Elements", M. Vogelius; SIAM Summer Meeting, University of Washington, Seattle, Washington, July 16-20, 1984.

18. "Implementation of a Finite Element Software System with h- and p-extension capabilities", by Barna A. Szabo, presented at the 8th Invitational Symposium on the Unification of Finite Element-Finite Differences and the Calculus of Variations, University of Connecticut, Storrs, Connecticut, May 3, 1985.

19. "Stress Singularities at Angular Corners of Composite Plates", Xing-Ren Ying And I. Norman Katz, SIAM Spring Meeting, Pittsburgh, PA, June 1985.

20. "An Overview of the p-version of the Finite Element Method," by I. Norman Katz, invited colloquium presentation at the Department of Mathematics, West Virginia University, Morgantown, West Virginia, June 27, 1985.

21. "A Global Algorithm for finding all roots of an Analytic Function in a bounded domain", Xing-ren Ying and I. Norman Katz, SIAM 1986 National Meeting, July 21-25, 1986, Boston, Massachusetts.

22. "A Reliable Argument Principle Algorithm to Find the number of zeros of an Analytic Function in a Bounded Domain", Xing-ren Ying and I. Norman Katz, Symposium on the Impact of Mathematical Analysis on the Solution of Engineering Problems, University of Maryland, September 17-19, 1986, College Park, Maryland.

END

4-87

DTIC